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The Trojan-Horse method for nuclear astrophysics

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Abstract. The Trojan-Horse method is an indirect approach to determine the low-energy astrophysical S-factor of direct nuclear reactions by studying closely related transfer reactions with three particles in the final state under quasi-free scattering conditions. The theoretical foundation and basic features of this approach are presented. General considerations for the application of method and two examples are discussed.

PACS. 24.50.+g Direct reactions – 25.70.Hi Transfer reactions

1 Introduction

Nuclear reaction rates are a basic ingredient in many astrophysical models that describe primordial nucleosynthesis, stellar evolution, supernovae etc. [1,2,3]. They have to be known with sufficient accuracy, *e.g.*, in the pp chains, CNO cycles, the s-, r-, p-, and rp-processes in order to describe quantitatively the observed abundance pattern of the elements. In principle it is preferable to measure the corresponding cross-sections directly in the laboratory but this is a very difficult task [4]. The cross-sections are often very small and in many reactions unstable nuclei are involved so that the experimental yields in direct experiments are very low. In the following only non-resonant charged-particle reactions are considered. In this case the repulsive Coulomb interaction leads to a strong suppression of the cross-section at small effective energies that are relevant to astrophysics. Usually the cross-section $\sigma(E)$ is measured at higher energies E and extrapolated to small energies with the help of the astrophysical S-factor

$$S(E) = \sigma(E) E \exp(2\pi\eta_{12}) \tag{1}$$

that exhibits only a weak energy dependence. The Sommerfeld parameter $\eta_{12} = Z_1 Z_2 e^2/(\hbar v_{12})$ depends on the charge numbers Z_1 and Z_2 of the two participating nuclei and their relative velocity v_{12} in the entrance channel of the reaction. But even if it is possible perform measurements directly at the relevant small energies in the laboratory the experimental cross-section $\sigma_{\text{exp}}(E)$ is enhanced as compared to the cross-section $\sigma_{\text{bare}}(E)$ of the bare nuclei due to the screening of the Coulomb potential by the electron cloud [5]. Quantitatively the electron screening if well described by

$$\sigma_{\text{exp}}(E) = \sigma_{\text{bare}}(E) \exp(\pi\eta U_e/E) \tag{2}$$

with the electron screening potential energy U_e. Unfortunately, screening potentials determined in direct experiments tend to be larger than their values expected from theoretical models. This discrepancy seems not to be fully understood and independent information on the screening effect is highly valuable. Additionally, in astrophysical applications the screening in the stellar plasma has to be accounted for.

As an alternative to the direct experiments indirect methods have been developed in recent years. They can give complementary information on the cross-sections that are relevant to astrophysics. The indirect approach depends on the particular type of reaction. A general characteristic of the indirect methods is that the astrophysically relevant two-body reaction is replaced by a three-body reaction at high energies. The relation of the cross-sections is established with the help of nuclear reaction theory that has to be well understood to give reliable information.

Well-known examples of indirect approaches are the Coulomb dissociation method [6,7] and the method of asymptotic normalization coefficients (ANC) [8,9] in order to determine the astrophysical S-factor of radiative capture cross-sections $a(b,\gamma)c$. In the Coulomb dissociation method the strong electromagnetic field of a highly charged target nucleus X serves as a source of equivalent photons and the photo dissociation cross-section $a(\gamma,c)b$, the inverse of the capture reaction, can be extracted from the cross-section of the Coulomb breakup reaction $X(a,bc)X$. In the ANC method the asymptotic normalization coefficient of the ground state wave function of nucleus c is extracted from transfer reactions. Then the relevant matrix elements for the radiative capture reaction can be calculated numerically.

For direct two-body reactions

$$A + x \longrightarrow C + c \tag{3}$$

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without a photon in the final state the Trojan-Horse (TH) method has been suggested as an alternative approach to determine indirectly the cross-section at low energies [10]. In this case the reaction (3) is replaced by a reaction

$$A + a \longrightarrow C + c + b \tag{4}$$

with three particles in the final state. The Trojan Horse $a = b + x$ is formed by attaching a spectator b to the nucleus x. The reaction (4) is studied under quasi-free scattering conditions where the momentum transfer to the spectator is small and other reaction mechanisms are suppressed. The relative energy in the system $A + a$ can be above or near the Coulomb barrier so that there are no suppression of the cross-section and no electron screening. Nevertheless, small relative energies in the system $A + x$ are accessible due to the particular kinematical conditions.

The validity of the TH method has been tested for various reactions by the Catania group of C. Spitaleri in recent years by comparing direct and indirect results under various kinematical conditions [11,12,13,14,15,16,17,18,19]. Basic theoretical considerations of the approach can be found in [20,21]. In the following the theoretical essentials of the method are presented and the application is discussed for two particular examples.

2 Theory of the Trojan-Horse method

The relation of the cross-sections for the two-body reaction (3) and the three-body reaction (4) is found by applying standard methods of direct-reaction theory. Denoting the system $C + c$ with B, the triple differential cross-section of reaction (4)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}E_{Cc}\,\mathrm{d}\Omega_{Cc}\,\mathrm{d}\Omega_{Bb}} = \frac{\mu_{Aa}\mu_{Bb}\mu_{Cc}}{(2\pi)^5\hbar^6}\frac{k_{Bb}k_{Cc}}{k_{Aa}} \times \frac{1}{2J_i+1}\sum_{M_i,M_f}|T_{fi}|^2 \tag{5}$$

is determined by the T-matrix element T_{fi} that contains all the relevant information. Here, reduced masses and wave numbers of system ij are denoted by μ_{ij} and k_{ij}, respectively. In the post-form distorted-wave Born approximation (DWBA) the T-matrix element is given by

$$T_{fi} = \left\langle \chi_{Bb}^{(-)}\phi_B\phi_b \middle| V_{xb} \middle| \chi_{Aa}^{(+)}\phi_A\phi_a \right\rangle \tag{6}$$

with distorted waves $\chi_{Aa}^{(+)}$, $\chi_{Bb}^{(-)}$ and bound state wave functions ϕ_A, ϕ_a, ϕ_b as in the case of usual transfer reactions. The wave function ϕ_B, however, is a complete scattering wave function $\Psi_{Cc}^{(-)}$ that contains the information on the two-body reaction. The potential V_{xb} is responsible for the binding of the nucleus x and the spectator b in the Trojan Horse a.

In the Trojan-Horse approach the full scattering wave function $\Psi_{Cc}^{(-)}$ is replaced by its asymptotic form for radii r larger than a strong absorption radius R. This so-called surface approximation is the essential approximation of the TH method. It is well justified since there is a strong suppression of the wave functions at smaller radii due to the absorptive part of the optical potentials in the entrance and exit channels of the three-body reaction. As a consequence of the surface approximation, the T-matrix element

$$T_{fi}^{\mathrm{TH}} = \frac{1}{2ik_{Cc}}\sqrt{\frac{v_{Cc}}{v_{Ax}}} \times \sum_l (2l+1)\left[S_{AxCc}^l U_l^{(+)} - \delta_{(Ax)(Cc)}U_l^{(-)}\right] \tag{7}$$

assumes a form similar to a scattering amplitude of a two-body reaction with the S-matrix elements S_{AxCc}^l of the reaction

$$C + c \longrightarrow A + x, \tag{8}$$

i.e. the inverse of the astrophysical important reaction (3). For simplification we assumed spinless nuclei in equation (7). The main difference, however, is the appearance of the factors $U_l^{(\pm)}(\boldsymbol{k}_{Bb}\boldsymbol{k}_{Cc}\boldsymbol{k}_{Aa})$ that in general are complicated reduced DWBA matrix elements. Their particular momentum dependence cancels the suppression of the S-matrix element

$$S_{AxCc}^l \propto \exp(-\pi\eta_{Ax}) \tag{9}$$

at low energies $E_{Ax} = \hbar^2 k_{Ax}^2/(2\mu_{Ax})$ due to the Coulomb interaction between A and x.

In order to find a simple physical interpretation further approximations can be applied that are, however, not necessary in the general approach. They only serve to show the features of the TH method more clearly. Using plane waves for the distorted waves $\chi_{Aa}^{(+)}$ and $\chi_{Bb}^{(-)}$ the cross-section (5) factorizes according to

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}E_{Cc}\,\mathrm{d}\Omega_{Cc}\,\mathrm{d}\Omega_{Bb}} = KF\ |W(\boldsymbol{Q}_{Bb})|^2\ \frac{\mathrm{d}\sigma^{\mathrm{TH}}}{\mathrm{d}\Omega} \tag{10}$$

with three contributions similar as in a plane-wave impulse approximation. The kinematic factor KF is proportional to k_{Ax}^{-3} at small energies E_{Ax}. The momentum amplitude $W(\boldsymbol{Q}_{Bb})$ is the Fourier transform of $V_{xb}\phi_a$ with respect to the relative coordination $\boldsymbol{r}_{xb}$. The argument

$$\boldsymbol{Q}_{Bb} = \boldsymbol{k}_{Bb} - \frac{m_b}{m_b+m_x}\boldsymbol{k}_{Aa} \tag{11}$$

corresponds to the momentum transfer to the spectator b. The TH cross-section

$$\frac{\mathrm{d}\sigma^{\mathrm{TH}}}{\mathrm{d}\Omega} = P\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \tag{12}$$

contains the usual cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$ of the two-body reaction (8) and a penetrability factor

$$P \propto k_{Ax}^3 \exp(2\pi\eta_{Ax}). \tag{13}$$

Collecting the k_{Ax}-dependent factors one immediately sees that the product $KF\,\mathrm{d}\sigma^{\mathrm{TH}}/\mathrm{d}\Omega$ is proportional to the astrophysical S-factor for $E_{Ax} \to 0$. It approaches a finite value in this limit. There is no suppression of the cross-section (10) due to the Coulomb barrier in the system Ax.

3 Application

In order to apply the Trojan-Horse method to a particular reaction (3) one has to select a Trojan Horse $a = b + x$ with a well-known ground state wave function that is highly clustered so that the momentum amplitude W for the breakup of a into b and c is well determined. Typical examples are the deuteron ${}^2\mathrm{H} = n + p$ and ${}^6\mathrm{Li} = \alpha + d$ that allow to study the transfer of nuclei that are the most relevant for nuclear astrophysics. The spectator b is usually not observed in the TH experiment since the complete kinematic information can be deduced from the momenta of the nuclei C and c that are detected in the final state and the known beam energy.

The width of the momentum distribution $|W|^2$ is related to the Fermi motion of the transferred particle x and the spectator b inside a with binding energy $\epsilon_a > 0$. The condition $\boldsymbol{Q}_{Bb} = 0$ defines the so-called quasi-free energy

$$E_{Ax}^{\mathrm{qf}} = E_{Aa}\left(1 - \frac{\mu_{Aa}\mu_{bx}^2}{\mu_{Bb}m_x^2}\right) - \epsilon_a \qquad (14)$$

in the Ax system. This relation is a purely kinematical consequence. The quasi-free energy (14) is much smaller than the relative energy E_{Aa} in the initial state of the three-body reaction (4) and easily falls into a range of energies of the two-body reaction (3) that are relevant for nuclear astrophysics. In an actual experiment a cutoff in the momentum transfer $\boldsymbol{Q}_{Bb}$ is chosen to emphasize the quasi-free reaction mechanism corresponding to the peak of the momentum distribution. This cutoff determines the range of accessible energies E_{Ax} around E_{Ax}^{qf}. Depending on the scattering angle in the two-body reaction (3) the quasi-free condition defines a pair of quasi-free angles where the particles C and c are detected in the laboratory.

Considering the approximations in the theoretical calculation one cannot expect that the absolute cross-section (10) is well determined quantitatively. However, the energy dependence is expected to be well reproduced. Therefore, the S-factor extracted in the TH method has to be normalized to known direct data at higher energies. In contrast to direct experiments, the main features of the TH method are that there are no electron screening and no suppression of the cross-section at small energies. It remains finite even in the limit $E_{Ax} \to 0$.

4 Examples

In order to check the validity of the Trojan-Horse method several reactions with stable nuclei under various conditions can be studied. *E.g.*, astrophysical S-factors are extracted in the TH method and compared to well-known data from direct experiments. Alternatively, cross-sections from TH experiments are compared with simulated cross-sections using information of direct experiments. Here we discuss examples of both approaches.

The cross-section of the reaction ${}^2\mathrm{H}({}^6\mathrm{Li},\alpha){}^4\mathrm{He}$ that is astrophysically relevant for the destruction of ${}^6\mathrm{Li}$ in the Big Bang has been measured to very low energies in a direct experiment with a deuterium gas target and a ${}^6\mathrm{Li}$ beam [22]. The cross-section is dominated by a non-resonant process with a s wave in the initial state. Extrapolating the experimental data (open circles in Figure 1) at higher energies with a polynomial fit to small energies a S-factor of $S(0) = 17.4\,\mathrm{MeVb}$ at zero energy has been extracted. From a comparison with the enhanced experimental data at very small energies below 100 keV an electron screening potential of $U_e = 330(120)\,\mathrm{eV}$ was deduced.

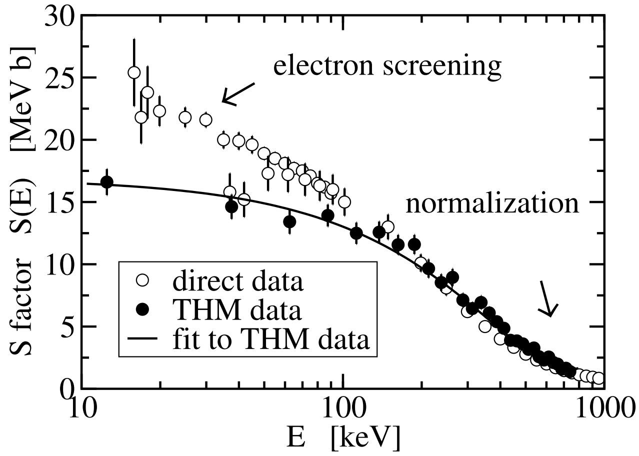

Fig. 1. Astrophysical S-factor $S(E)$ for the reaction ${}^2\mathrm{H}({}^6\mathrm{Li},\alpha){}^4\mathrm{He}$ from a direct experiment [22] (open circles) and from the TH method (filled circles).

In a corresponding Trojan-Horse experiment using the reaction ${}^6\mathrm{Li}({}^6\mathrm{Li},\alpha\alpha){}^4\mathrm{He}$ the nucleus ${}^6\mathrm{Li}$ has been chosen as the Trojan horse with an α particle as the spectator [17,18]. With a beam energy of 6 MeV a quasifree energy $E^{\mathrm{qf}} = 25\,\mathrm{keV}$ could be reached for the two-body reaction. Due to the symmetry of the TH reaction both the projectile and the target were used as Trojan Horses. The S-factor was extracted from the experimental data with a cutoff of $35\,\mathrm{MeV}/c$ in the momentum distribution. Absolute values were derived by normalizing to the direct data for energies above 600 keV (see fig. 1). A polynomial fit to the indirect data yields a S-factor at zero energy of $S(0) = 16.9(0.5)\,\mathrm{MeVb}$ consistent with the direct experiment. Comparing the S-factor from the TH experiment with the direct data an electron screening potential of $U_e = 320(50)\,\mathrm{eV}$ was determined, supporting the value extracted from the direct experiment alone. Both values are substantially larger than the adiabatic limit of 186 eV expected from theory.

Differential cross-sections of the reaction ${}^6\mathrm{Li}(p,\alpha){}^3\mathrm{He}$ for a large range of energies were measured in a direct experiment [23]. From them the coefficient B_l in the angular expansion of the cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_l B_l P_l(\cos\theta) \qquad (15)$$

with Legendre polynomials P_l were extracted. The coefficients B_l are well described in a R-matrix fit (see fig. 2).

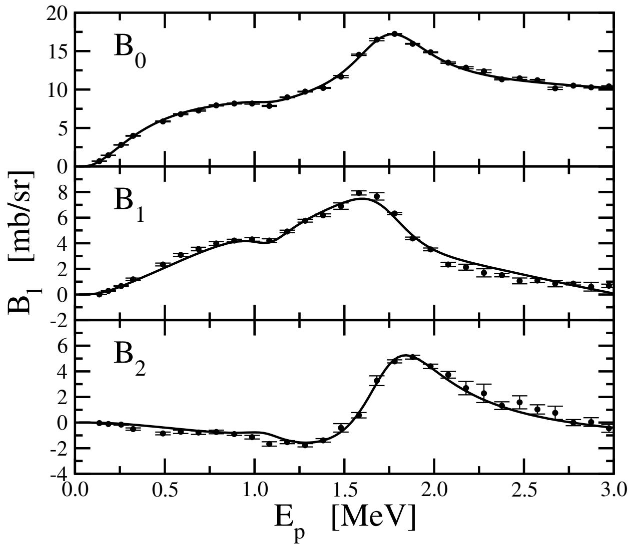

Fig. 2. Coefficients B_l in the expansion (15) as a function of the proton energy E_p for the reaction $^6\mathrm{Li}(p,\alpha)^3\mathrm{He}$ in an R-matrix fit to experimental data [23].

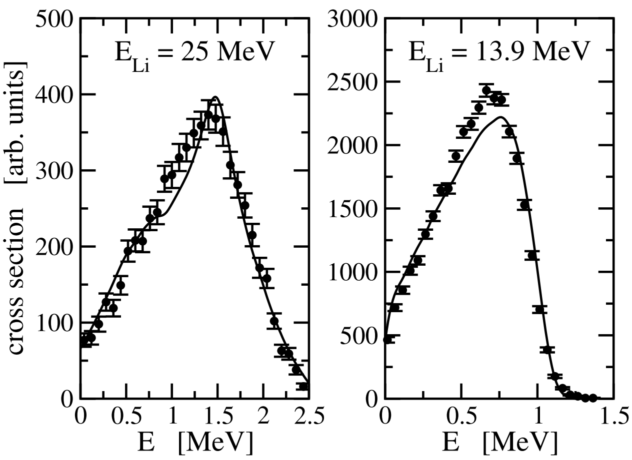

Fig. 3. Cross-sections of the Trojan-Horse reaction $^2\mathrm{H}(^6\mathrm{Li},\alpha^3\mathrm{He})n$ ([19] and preliminary data) compared with a simulation using S-matrix elements from a R-matrix fit to direct data.

There are both nonresonant s-wave and resonant p-wave contributions. The S-matrix elements derived from the R-matrix fit were then used in the simulation of Trojan-Horse experiments.

Choosing the deuteron as the Trojan Horse, experiments with the reaction $^2\mathrm{H}(^6\mathrm{Li},\alpha^3\mathrm{He})n$ were performed with the neutron as the spectator [19]. Energies of 13.9 MeV and 25 MeV for the ^{6}Li beam were selected that correspond to quasi-free energies E^{qf} of -0.24 MeV and 1.35 MeV, respectively. The experimental cross-sections as a function of the relative energy in the $^6\mathrm{Li}+p$ system are compared in fig. 3 with the simulation applying a cutoff of 30 MeV$/c$ in the momentum transfer to the spectator. The drop of the cross-section above 1.5 MeV and 1.0 MeV, respectively, is a consequence of this momentum cutoff. The overall shape of the cross-sections is well reproduced by the simulation, however, small discrepancies remain that have to be investigated in more detail.

5 Summary and outlook

The Trojan-Horse method allows to extract the energy-dependence of the astrophysical S-factor of a direct two-body reaction from the measurement of a related transfer reaction under quasi-free scattering conditions. The relation of the cross-sections is found by applying a distorted-wave Born approximation with the surface approximation that is essential for the method. A characteristic feature of this indirect approach is that the cross-section at low energies is not suppressed due to the Coulomb barrier in the two-body system. Additionally, there is no electron screening and information on the electron screening potential can be extracted by comparing to data of direct measurements. First applications of the method to well-studied two-body reactions with simple theoretical approximations were quite successful so far, however, further experimental tests are necessary to establish the validity of the method. The full theory has not been applied yet and more elaborate calculations are required. A comparison of different approximations will give additional information on the accuracy and applicability of the method.

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